Exercise 22

Given the functions $f(x) = \frac{1-x}{x}$ and $g(x) = \frac{1}{1+x^2}$, find the following:

(a) $(g \circ f)(x)$

(b) $(g \circ f)(2)$

Solution

Compute $(g \circ f)(x)$ by plugging the formula for $f(x)$ where x is in the formula for $g(x)$.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \frac{1}{1 + \left(\frac{1-x}{x}\right)^2} \\ &= \frac{1}{1 + \frac{(1-x)^2}{x^2}} \\ &= \frac{1}{1 + \frac{(1-x)^2}{x^2}} \cdot \frac{x^2}{x^2} \\ &= \frac{x^2}{x^2 + (1-x)^2} \\ &= \frac{x^2}{x^2 + (1 - 2x + x^2)} \\ &= \frac{x^2}{2x^2 - 2x + 1}\end{aligned}$$

Therefore,

$$(g \circ f)(2) = \frac{(2)^2}{2(2)^2 - 2(2) + 1} = \frac{4}{2(4) - 4 + 1} = \frac{4}{5}.$$